Natural convection experiments in a horizontal porous layer saturated with cold water

T. L. Spatz and D. Poulikakos
Department of Mechanical Engineering, University of Illinois at Chicago, Chicago, IL 60680, USA

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This study reports results from a series of laboratory experiments focusing on the problem of high Rayleigh number natural convection in a horizontal water-saturated porous layer consisting of packed glass spheres. The layer is heated from below and cooled from above. The main novel feature of the study is that it documents the effect of the density extremum of water, corresponding to the temperature of about 4°C at atmospheric pressure, on the temperature field and on the overall heat transfer through the layer. It is found that the presence of the density extremum seriously suppresses the heat transfer process. The value of the conduction referenced Nusselt number depends both on the Rayleigh number and on the relative magnitude of the cold top wall temperature and the temperature corresponding to the density extremum. A general correlation is proposed for the Nusselt number, $Nu = 0.0045[Ra_p R^q]^{1.19}$. This correlation is applicable for all values of the top wall temperature in the vicinity of the temperature of the density extremum.

Keywords: experimental natural convection; bed of glass spheres; density extremum of water

Introduction

Natural convection of water in a porous medium near the freezing point of water has diverse engineering applications ranging from ice slurries, the melting of permafrost, ice storage systems and artificial soil freezing for mining and construction purposes. Unlike most fluids commonly used in engineering applications which possess monotonic, almost linear for small temperature ranges, density-temperature relationships, water possesses a density maximum at 4°C at atmospheric pressure. The presence of this density maximum greatly affects water natural convection and yields flows markedly different from those of other fluids or even water for temperature ranges away from its density maximum.

Of the several alternative relationships proposed and used by several investigators to describe the dependence of water density on temperature in the vicinity of the density maximum, for the present study we chose to adopt the most recent one proposed by Gebhart and Mollendorf.[1] The reason for this choice is that the above mentioned relationship holds for a rather large temperature range (between 0 and 20°C), it is simple and it appears to be at least as accurate as any other density-temperature relationship for the region of the density maximum of water.[2–4] This relationship reads

$$\rho = \rho_M[1 - \alpha_1|T - T_M|^q] \tag{1}$$

where subscript M denotes the density maximum, $T_M = 4.02932°C$, $\rho_M = 999.972\ kg/m^3$, $\alpha_1 = 9.297173 \times 10^{-6}\ (°C)^{-q}$, and $q = 1.894816$. Equation 1 is plotted diagrammatically in Figure 1 to illustrate clearly the presence of the density extremum.

The experimental and numerical investigation of natural convection of water saturating a horizontal porous layer has attracted the attention of several investigators. Masuoka[5] in the early seventies was the first to report experimental and analytical results for the onset of convection in a water-saturated layer of spherical beads. He reported correlations for the Nusselt number for low and high values of the Rayleigh number. However, both the theoretical and the experimental studies did not account for the effect of the density extremum. Experiments on natural convection in a porous layer saturated by cold water were conducted by Yen.[3] In correlating his results, Yen[3] used a third-order polynomial density-temperature relationship to account for the density extremum. He focused on low and moderate values of the Rayleigh number and reported limited results on the effect of the density extremum on the overall heat flux through the layer. He found, nevertheless, that this effect was significant. A linear stability analysis for the same problem was reported by Sun *et al.*[6] The main contribution of this study was the determination of the critical values of Rayleigh number marking the onset of convection. It was found that the presence of the density extremum significantly affects the value of the critical Rayleigh number and, therefore, the onset of convection in a porous layer saturated by cold water and heated from below.

More recently, Blake *et al.*[7] published numerical simulations of two-dimensional natural convection near 4°C in a water-

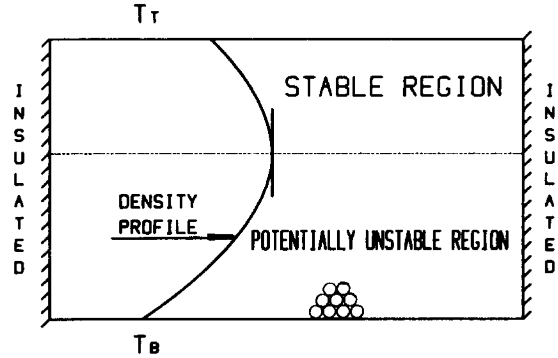

Figure 1 Diagrammatic representation of the system of interest: A horizontal porous layer saturated with water in the vicinity of its density extremum

saturated porous layer. The flow pattern in the layer was multicellular and depended on the values of the horizontal wall temperatures, the layer geometric aspect ratio and the Rayleigh number. The numerical results agreed well with the linear stability analysis results in Ref. 6 regarding the critical values of the Rayleigh number marking the onset of convection.

The present paper focuses on the high Rayleigh number regime and reports new experimental results for the temperature field and heat transfer inside a horizontal layer of glass beads saturated with cold water. A general correlation is proposed documenting quantitatively the effect of the density extremum on the overall heat transfer across the layer. This correlation is accurate regardless of the relative magnitude of the temperatures of the horizontal walls and the density extremum temperature.

Apparatus and procedure

The setup for the experiment consisted of the test apparatus and three supporting devices. The three supporting devices were a data acquisition system, a power supply, and a bath refrigerator circulator. The data acquisition system consisted of a Hewlett-Packard 150 Touchscreen II PC, two Hewlett-Packard 3421A data acquisition/control units, a printer, and a Hewlett-Packard data acquisition software package. The power supply for the heater element was a Hewlett-Packard 6012B 1000-W dc power supply. A Brinkmann RC20 refrigerator unit was used to cool the top plate of the apparatus.

The internal dimensions of the test apparatus (Figure 2) measured 48.3 cm long by 25.4 cm tall by 12.7 cm deep. Plexiglas (low thermal conductivity) 1.27 cm thick was used to construct the side walls of the apparatus so that these walls did not alter the vertical heat transport. The top and the bottom walls were constructed out of aluminum of thickness 2.54 cm and 1.27 cm, respectively. The bottom plate had 10 (T-type) thermocouples embedded 1.6 mm from the inside surface. Machined into the bottom plate was a cavity for a 3.2-mm-thick, high-density (10 W/in^2) flexible rubber electric heater. Highly conductive silicon paste was used to insure good conductivity between the heater and the aluminum plate. The heater was insulated from below with two layers of 8-mm-thick asbestos gasket material and a layer of 16-mm-thick rubber insulation. The top plate was machined to allow for 8 thermocouples, 21 access ports, and a counterflow heat exchanger. The counterflow heat exchanger was constructed by milling four channels into the top plate lengthwise. A 50% ethylene glycol/water solution precooled by the bath refrigerator was circulated through the heat exchanger. The direction of flow of the coolant was alternated in adjacent channels to establish isothermality (within ± 0.5°C) along the top plate. The channels were sealed with a 1.27-cm-thick sheet of Plexiglas which covered the entire top plate.

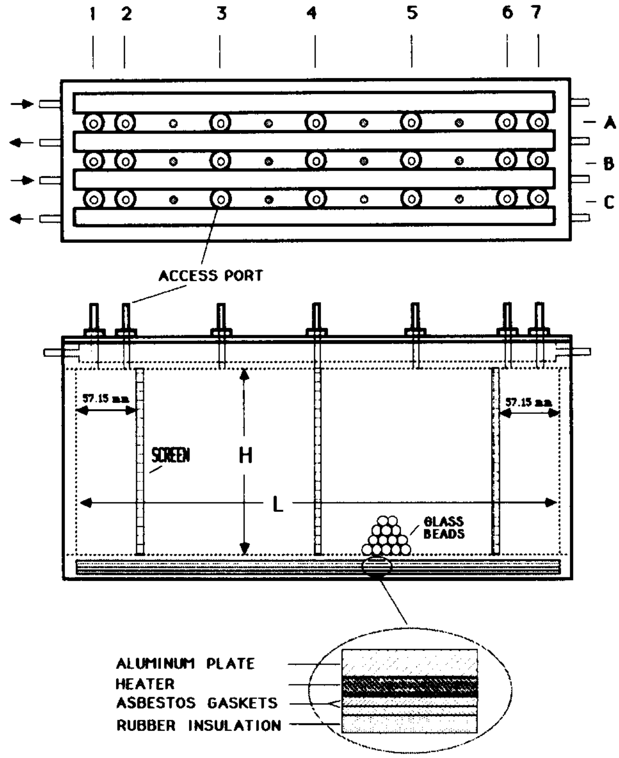

Figure 2 The experimental apparatus: (a) front view; (b) top view

The porous material was made up of glass spheres (diameter of 5 mm) completely filling the test apparatus. Placed within the porous material were three vertical plastic screens, each

Notation

A	Area of heated surface, m^2
d	Bead diameter, m
g	Gravitational acceleration, m/s^2
H	Enclosure height, m
k	Thermal conductivity, W/m K
K	Permeability, Equation 3
Nu	Nusselt number, Equation 5
q	Exponent of density-temperature relationship
Q	Power input to the heater, W
R	Dimensionless parameter
Ra_p	Darcy-modified Rayleigh number for cold water natural convection, Equation 2
Ra*	Rayleigh number, $\mathrm{Ra}^* = \frac{g\beta}{\alpha\nu}(\Delta T)H^3$
T	Temperature, °C
T_B	Bottom wall temperature, °C
T_M	Density extremum temperature, °C
T_T	Top wall temperature, °C
ΔT	Bottom to top temperature difference, °C

Greek symbols

α	Thermal diffusivity, m^2/s
α_1	Coefficient of density-temperature relationship
α_p	Porous medium effective thermal diffusivity, $k_p/(\rho C_p)_f$
β	Coefficient of volumetric thermal expansion, K^{-1}
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
ρ_M	Extremum density, kg/m^3
ε	Porosity
θ	$(T-T_T)/(T_B-T_T)$

Subscripts

B	Bottom (hot) wall
f	Fluid (water)
M	Value corresponding to density extremum
p	Porous medium
s	Solid (glass)
T	Top (cold) wall

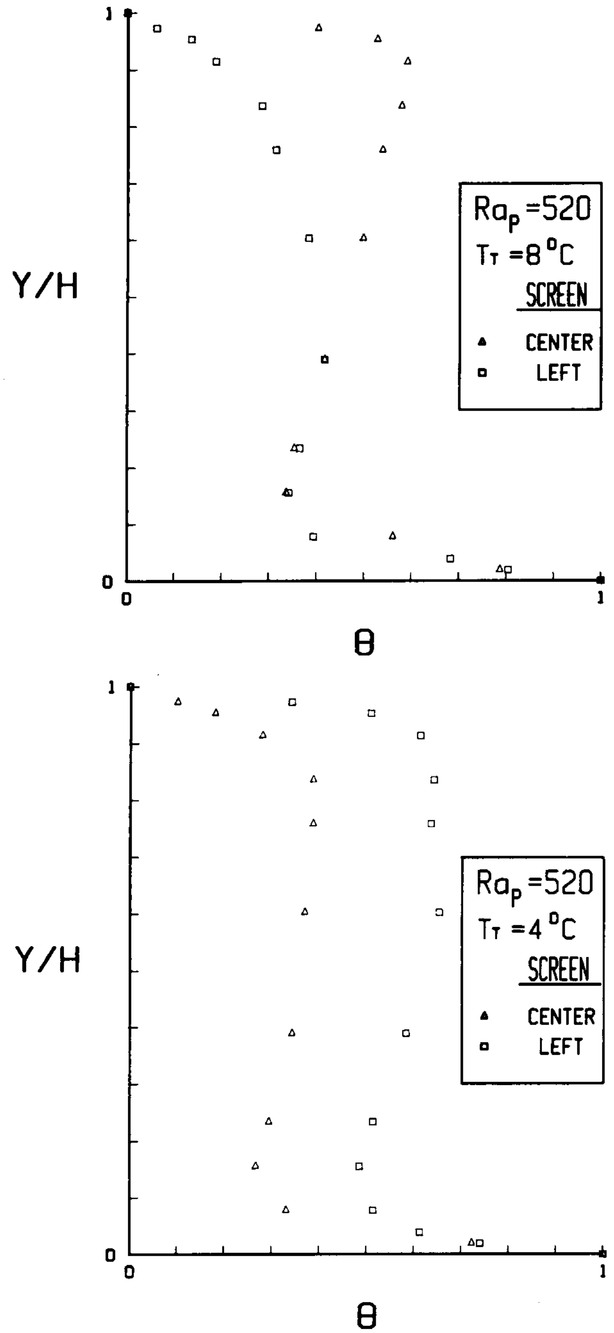

Figure 3 Temperature profiles for $Ra_p = 520$ and different top wall temperatures: (a) $T_T = 8°C$; (b) $T_T = 4°C$

containing 12 (T-type) thermocouples. One screen was placed in the center of the tank and the other two were placed 5.7 cm from each side wall.

The apparatus was mounted on a 10-cm-thick hardwood table and was insulated on the top and all sides by a 2.54-cm-thick layer of K-fac # 19 insulating board. Finally, the apparatus was filled with distilled water.

At least 10 steady-state experiments were run for each of the three different top plate temperatures; 0, 4, and 8°C. For each top plate temperature the power input to the heater was varied between 2 and 100 W in varying steps so that the bottom wall temperature ranged from about 13 to 30°C. Between each setting, steady state was allowed to occur and top and bottom plate temperatures were recorded. Complete temperature profiles for all steady states were obtained from the three thermocouple screens. These profiles allowed for the determination of the temperature field in the cavity.

Qualitative visualization of the flow field was accomplished by adding potassium permangenate crystals to the fluid through some of the access ports. A high-intensity light was placed on the opposite side of the apparatus from the observer. As the crystals dissolved, they dyed the water purple to give an indication of the general flow pattern within the cavity.

Results and discussion

Temperature profiles in the vertical direction inside the enclosure for representative steady states ($Ra_p = 520$, $T_T = 0$, 4, 8°C) are shown in Figure 3(a), (b). It is worth clarifying that the Raleigh number is appropriate for natural convection of cold water and is based on the enclosure height

$$Ra_p = \frac{g\alpha_1}{\alpha_p \nu} (\Delta T)^q K H \tag{2}$$

where the permeability K for a porous matrix of porosity ε consisting of glass spheres of diameter d is given by

$$K = \frac{d^2}{150}\left[\frac{\varepsilon^2}{(1-\varepsilon)^2}\right] \tag{3}$$

The effective thermal diffusivity, α_p, in Equation 2 is the ratio of the effective thermal conductivity of the porous medium to the thermal capacity of the fluid as defined in the nomenclature. The effective thermal conductivity of the porous medium is the volume–average thermal conductivity of the fluid and the porous matrix

$$k_p = \varepsilon k_f + (1-\varepsilon) k_s \tag{4}$$

The temperature profiles were obtained using two of the the thermocouple screens discussed earlier. These screens are the center and the left screens (Figure 2). The results reported in Figure 3 indicate that the temperature field inside the cavity is rather complex and depends on the relative magnitude of the top wall temperature and the density extremum temperature. A consistent feature of the temperature field is that it consists of two thermal boundary layers along the two horizontal walls surrounding a thermally stratified core region. The sharpness of the thermal boundary layers as well as the degree of stratification of the core region depends on the value of the top wall temperature. Note that the value of the top wall temperature relative to the density extremum temperature indicates what portion of the fluid in the cavity is stably stratified (Figure 1).

Focusing on Figure 3(a) we realize that the effect of the density extremum, which becomes stronger as the top wall temperature changes from 8 to 4°C, is to reduce the heat transfer through the cavity. Comparing Figure 3(a) to Figure 3(b) indicates that for $T_T = 4°C$ the temperatures at the centerline screen location are, overall, lower than the temperatures at the left side screen location. However, for $T_T = 8°C$ the opposite is true. This interesting observation is a direct result of the fact that the direction of rotation of the cells changes depending on the top wall temperature. Our flow visualization indicated that in all cases the flow pattern in the enclosure consisted of four cells. For $T_T = 8°C$ the flow direction was such that an upward "warm" plume existed at the centerline. On the other hand, for $T_T = 0$ and 4°C a downward "cold" plume existed along the centerline. In addition, the size of the cells depended on T_T. Because of the limitations and the qualitative nature of our flow visualization method, no conclusive results on the flow

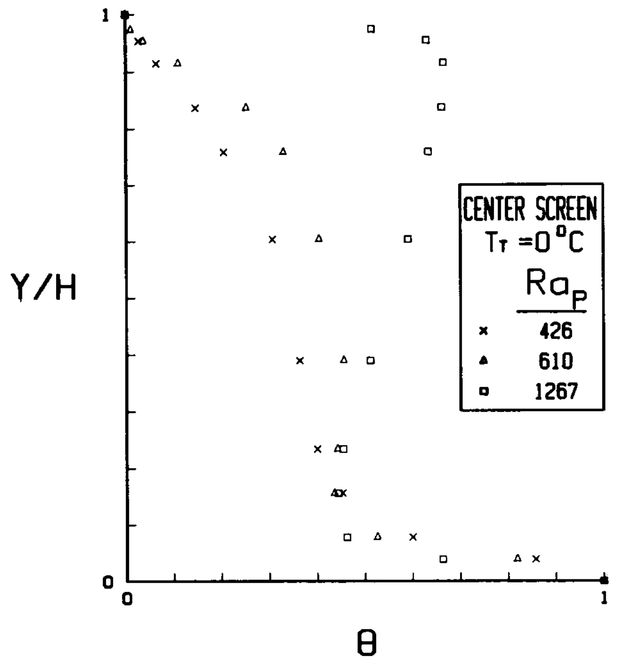

Figure 4 Temperature profiles for $T_T=0°C$ at the centerline for three different Rayleigh numbers ($Ra_p=426$, 611, 824)

field were obtained. It is felt that accurate flow visualization methods (nonexistent presently to the best of our knowledge) need to be developed to enhance our knowledge of flow patterns within opaque or translucent porous matrixes. It is worth noting that the flow pattern observed in the cavity agrees qualitatively with the four-cell pattern predicted numerically by Blake *et al.*[8] for a similar aspect ratio and Rayleigh number.

In Figure 4, the effect of Ra_p on the temperature profiles is illustrated for a case where the density extremum effect is present ($T_T=0°C$). Increasing Ra_p yields two sharp boundary layers along the two horizontal walls and, therefore, enhances the convection contribution to the overall heat transfer. This fact is more obvious in the temperature profiles corresponding to the highest of the three Rayleigh numbers in Figure 4.

Next, attention is shifted to reporting heat transfer results. This is achieved with the help of the conduction referenced Nusselt number defined as

$$Nu=\frac{Q}{k_p A(\Delta T/H)} \tag{5}$$

All the symbols in the above equation are defined in the notation. To establish the credibility of our heat transfer results, a comparison is made in Figure 6 between the present findings for $T_T=8°C$ (where the density extremum effect is minimal) and the findings of Masuoka[5] for natural convection in a porous layer in the absence of the density extremum effect. Clearly, the agreement is very good especially if one takes into account the fact that since Masuoka[5] reports his results only graphically, we obtained the equation of the solid line in Figure 5 by reading the data out of a rather small size Figure in Ref. 5. The equation of the solid line in Figure 5 obtained from Masuoka's results is

$$Nu=0.02Ra^*\left(\frac{K}{H^2}\right) \tag{6}$$

Figure 6 shows that for all values of T_T, increasing Ra_p increases Nu. In addition, the drastic effect of the density extremum on the overall heat transfer in the cavity is obvious. If the temperature of the top wall decreases below 8°C, the value of Nu decreases significantly (for a fixed value of Ra_p). For example, for $Ra_p=400$, going from $T_T=8°C$ to $T_T=0°C$ reduces the value of Nu down to one third (from $Nu\simeq 9$ to $Nu\simeq 3$). This is an important result, especially if it is recalled that in the absence of the density extremum effect Nu depends only on Ra_p and is independent of the actual value of T_T. The density extremum effect is most severe for $T_T=0°C$. Since this is the freezing temperature of water the importance of the density extremum effect on natural convection flows in freezing or melting applications involving water is paramount.

Also shown in Figure 6 are the approximate values for Ra_p above which the rotational direction of the cells in the cavity is reversed. These values are denoted by small vertical lines one for each top wall temperature. The rotational direction of the flow field for values of Ra_p to the left of these lines is opposite to the rotational direction of the flow field for values of Ra_p to the right of these lines. The dependence of the rotational direction of the flow field in a horizontal porous layer on the value of the Rayleigh number has also been discussed by Poulikakos[8] who reported results from a series of numerical simulations on natural convection in superposed porous and fluid layers.

From an engineering standpoint it is useful to produce a correlation for Nu, valid for all values of the top wall temperature. This is achieved by introducing a new dimensionless

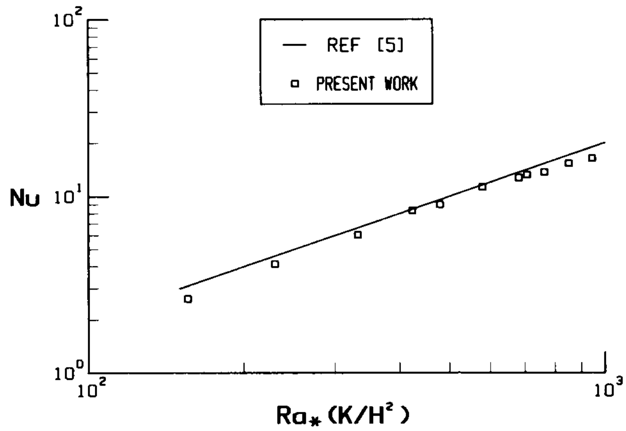

Figure 5 Nu versus $Ra^*(K/H^2)$: comparison of experimental data with the correlation of Masuoka[5]

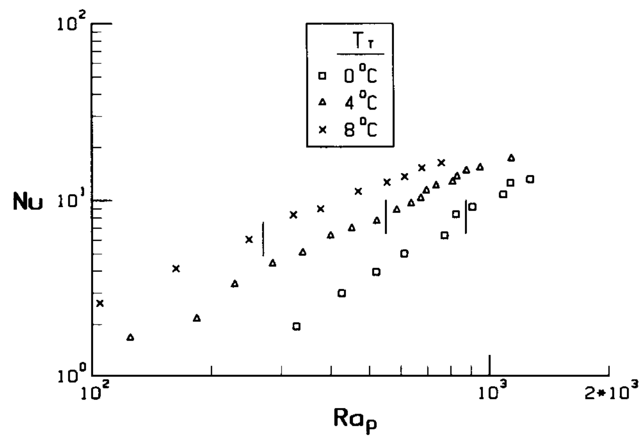

Figure 6 Nu versus Ra for different top wall temperatures: 0, 4, and 8°C

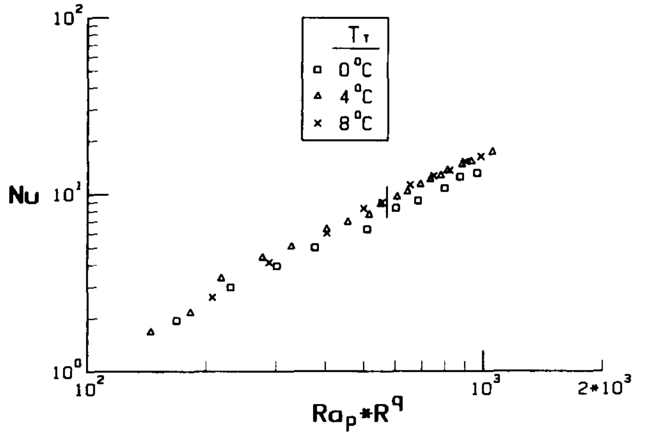

Figure 7 The proposed general correlation for the heat transfer results

parameter R defined as

$$R=\frac{T_M-T_B}{T_T-T_B} \tag{7}$$

For fixed values of the bottom and top wall temperatures, parameter R gives an indication of the portion of the temperature field in the enclosure that is potentially unstable. Figure 7 shows the data of Figure 6 but now plotted as Nu versus $Ra_p \times R^q$. Clearly, the data in Figure 7 can be correlated as a single line

$$Nu=0.0045[Ra_pR^q]^{1.19} \qquad (100<Ra_p<1000) \tag{8}$$

Correlation (8) is valid for all values of the top plate temperature. Note that the group $Ra_p \times R^q$ represents an effective Rayleigh number which is based only on the unstable portion (T_B-T_M) of the temperature difference across the enclosure (T_B-T_T).

Conclusions

This paper reported the main results of an experimental study on the effect of the density extremum of water, occurring in the vicinity of 4°C at atmosphere pressure, on high Rayleigh number natural convection in a horizontal water-saturated porous layer. The porous layer consisted of glass beads and it was cooled from above and heated on the underside. The temperatures of both horizontal walls were held constant. The flow inside the layer was multicellular. The direction of rotation of the cells depended both on the Rayleigh number and on the value of the top wall temperature, relative to the density extremum temperature.

It was found that the presence of the density extremum drastically suppressed the overall heat transfer through the system. For the most part, the temperature field in the enclosure consisted of two thermal boundary layers along the two horizontal walls, surrounding a stratified core region. Finally, a general correlation was obtained, Equation 8, for the Nusselt number. This correlation is valid for all values of the top wall temperature in the vicinity of the density extremum temperature, and can be used to estimate the overall heat transfer through the porous layer when the flow is affected by the presence of the density extremum.

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